

Modal Analysis with ArtemiS SUITE

Description of the method used and application examples

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ABOUT THIS DOCUMENT

Topic / Starting point

The evaluation of transfer functions measured in the course of an experimental modal analysis requires special analysis software. This software can be used to determine the modal parameters of test structures and to obtain important information on their dynamic behavior. The software applications are based on various calculation methods.

Content

This document provides

- a short introduction to modal analysis
- background information on the calculation method used in the ArtemiS SUITE Modal Analysis Project and its implementation
- application examples that provide an insight into the operation and potential of the algorithm used

Target group

The document is particularly intended for readers who are interested in learning more about the technical background and the possibilities of modal analysis with ArtemiS SUITE. The description of the available tools and their application can be used to evaluate the benefit for one's own task and the applicability in everyday work.

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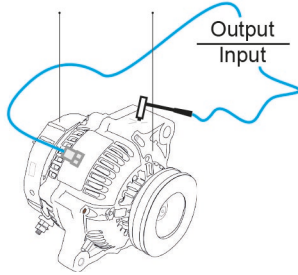
1 INTRODUCTION

Experimental Modal Analysis

The aim of the Experimental Modal Analysis (EMA) is to determine the structural dynamic properties of a real structure. This can be used, for example,

- to find the cause of sound and vibration problems and to optimize the transmission paths
- to localize weak points of a structure with respect to dynamic loads and evaluate different approaches on how to remedy them
- to validate or parametrize numerical calculation methods

Determination of the modal parameters



The basis for modal analysis is the set of measured transfer functions. A transfer function describes the measured relationship between an excitation force and the vibration it causes at a point of the structure. The modal parameters are then extracted from the transfer functions: the natural frequencies, the modal damping and the corresponding mode shapes. In combination, these parameters provide a deeper understanding of the structural dynamic behavior.

The LSCF algorithm

Many different algorithms were developed in the past to assist the user in extracting modal parameters. The Modal Analysis Project in ArtemiS SUITE uses an implementation based on the Least-Squares-Complex-Frequency method (LSCF method). Figure 1 shows the role of the LSCF method within modal analysis. The LSCF method requires the measured transfer functions as input data. In addition, suitable setting parameters for the LSCF method must be defined to achieve an optimal result. The output parameters are the so-called poles, which can be used to determine the modal parameters (natural frequencies, modal dampings and mode shapes).

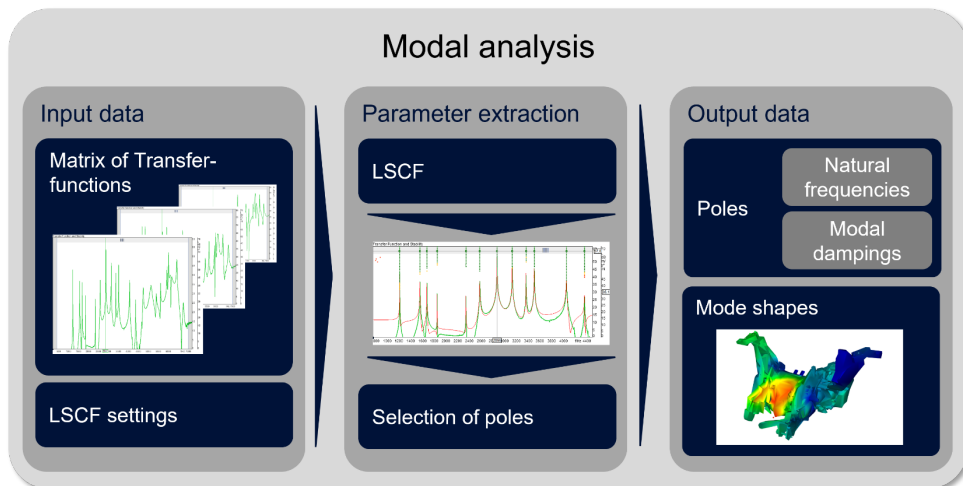


Figure 1: Overview of the LSCF method and its role in modal analysis

The following chapter explains in excerpts the theoretical background of the LSCF method, its setting parameters and its output quantities. The third chapter supplements the theoretical explanations with application examples from the user's point of view.

*Advantages of the LSCF-
algorithm*

Selecting the LSCF method as the basis for implementation in the ArtemiS SUITE Modal Analysis Project was based, among other things, on its versatility. It is suitable, for example, for

- applications with a wide frequency range
- structures with poles that lie close together in the transfer functions
- a wide damping range

Due to its versatility, the LSCF method provides good results for a wide range of applications. There are special algorithms for some applications that could provide even more accurate results, but to use them, the user must have extensive specialized knowledge to select and correctly parametrize the algorithm that is exactly the right one for their respective application. The versatility of the implemented method offers many advantages, especially for users who are not familiar with the various different algorithms and their advantages and disadvantages. In addition, it is particularly efficient, because it has been optimized with regard to both memory requirements and computing time.

2 PARAMETER EXTRACTION: LEAST-SQUARES-COMPLEX-FREQUENCY METHOD (LSCF)

Development of the LSCF method

Initially, the Least-Squares-Complex-Frequency method (LSCF method) was developed to calculate the initial values for another pole extraction algorithm [2]. It soon turned out, however, that the poles determined by the LSCF method were already quite accurate and did not require further optimization. Since that time, the LSCF method and its multi-reference implementation (Poly LSCF) presented in [1] have established standards in terms of robustness and speed and have been used in a variety of modal analysis programs with slight modifications and different names.

The following is a simplified presentation of the basic features of the algorithm. Please refer to more detailed literature for complete information on the implementation [1], [3].

Matrix of the transfer functions

The ratio of excitations and responses is described by the matrix of the measured transfer functions $\hat{H}(\omega) \in \mathbb{C}^{N_o \times N_I}$ with N_o = number of responses and N_I = number of excitations. It is assumed that the matrix of the transfer functions can be approximated by the following equation:

$$\hat{H}(\omega) = N(\omega) \cdot D^{-1}(\omega)$$

In this case, $N(\omega) \in \mathbb{C}^{N_o \times N_I}$ is called the numerator matrix polynomial and $D(\omega) \in \mathbb{C}^{N_I \times N_I}$ the denominator matrix polynomial. The following applies to each row of the matrix of transfer functions:

$$\hat{H}_i(\omega) = N_i(\omega) \cdot D^{-1}(\omega) \quad i = 1, 2, \dots, N_o$$

Calculation of the numerator polynomial

The numerator polynomial is described by the following formula:

$$N_i(\omega) = \sum_{j=0}^n B_{ij} \Omega_j(\omega) \quad B_{ij} \in \mathbb{R}^{1 \times N_I}$$

While n denotes the model size and thus is an input value of the algorithm, B_{ij} are the polynomial coefficients of the numerator polynomial. The larger the model size chosen, the more degrees of freedom the modal model has. Each degree of freedom allows to approximate/detect a resonance in the transfer functions. If the model size is chosen too small, it prevents the modal model from approximating the system with sufficient accuracy. On the other hand, a large model size results in a longer computation time, since a large model size increases the size of the matrices used in the algorithm.

In the equation above, Ω denotes the polynomial basis function:

$$\Omega_j(\omega) = e^{j\omega\Delta t}$$

Calculation of the denominator polynomial

The denominator polynomial has the following form:

$$D(\omega) = \sum_{j=0}^n A_j \Omega_j(\omega) \quad A_j \in \mathbb{R}^{N_I \times N_I}$$

Here, A_j are the polynomial coefficients of the denominator polynomial. A linearized Least Squares Estimation leads to the polynomial coefficients.

Using the coefficients of the denominator polynomial A_j , the so-called Companion Matrix can be formed:

Companion Matrix

$$A_C = \begin{bmatrix} A'_{n-1} & \dots & A'_1 & A'_0 \\ I & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & I & 0 \end{bmatrix} \quad \begin{aligned} A'_j &= -A_n^{-1}A_j \\ I &= \text{Identity matrix} \end{aligned}$$

The eigenvalues of the companion matrix are called the poles of the modal model. The eigenvectors are used to derive the so-called modal participation factors. The poles are related to natural frequency and damping as follows:

$$\lambda_r, \lambda_r^* = -\xi_r \omega_r \pm j \omega_r \sqrt{1 - \xi_r^2} \quad \begin{aligned} * &= \text{conjugate complex} \\ \xi &= \text{damping ratio} \\ \omega_r &= \text{eigen angular frequency [rad/s]} \end{aligned}$$

Stability plot

The calculation is performed iteratively with an increasing model size n until the maximum model size defined by the user is reached. Afterwards, the stability plot (see **Figure 7**) is used to determine how the poles behave over the iterations and to what extent the results fluctuate over the iterations. Besides fundamental inaccuracies due to the quality of the measured data, these fluctuations in the identified poles can be attributed to an insufficient model size. With too small a model size, the algorithm has too few degrees of freedom to identify all poles. In this case, the identified poles have to compensate the influences of the unidentified poles and thus show wrong values which are subject to strong fluctuations. From the iteration where all physically existing poles are actually found, the fluctuation in the parameters of the poles decreases and they are identified as stable with suitably chosen boundary conditions (compare **page 9**).

A minimum number of “similar” poles is required as a boundary condition of the selection process. If this number is reached, a pole is considered stable. In addition, similarity criteria for the damping ratio and the frequency must be specified. These criteria are used to select the poles.

Approximation of the transfer functions

Knowing the poles, the measured transfer functions can be approximated by the following formula.

$$\begin{bmatrix} H_1(\omega) \\ \vdots \\ H_{N_O}(\omega) \end{bmatrix} = \sum_{r=1}^{N_m} \left(\frac{\Phi_r L_r^T}{j\omega - \lambda_r} + \frac{\Phi_r^* L_r^H}{j\omega - \lambda_r^*} \right) - \frac{LR}{\omega^2} + UR$$

LR (Lower Residual) and UR (Upper Residual) include the influences of modes outside the limits of the considered frequency range. With the poles λ_r and the modal participation factors L_r being known from the eigenvalues and eigenvectors of the companion matrix, the eigenmodes Φ_r as well as the lower and upper residuals are the only unknowns and can be determined using the least-squares approach. Thus, the modal model of the measured structure including all modal parameters (natural frequencies, damping ratios, eigenmodes) is derived.

3 PRACTICAL APPLICATION

EXAMPLE 1: TORQUE ARM OF AN ELECTRIC VEHICLE (EV)

The following describes the experimental modal analysis of the torque arm of an EV using ArtemiS SUITE. This analysis aims at validating the results of a numerical modal analysis. For this purpose, different tools are used:

- Measurement Point Library
- Impact Measurement
- Modal Analysis Project
- Shape Comparison Project



Measurement setup torque arm

For the measurements of the transfer functions, the torque arm is removed and suspended from a rubber cord with the aid of a nylon thread glued to it in order to



Figure 2: Measurement set-up torque arm

approximate free vibration behavior. The measurements are carried out with the „Roving Hammer“ method: To do this, the structure is struck with a hammer at various positions. The structural response is measured with an accelerometer at a fixed position. This has the advantage that the mass influence of the sensors does not change between the individual measurements.

Measurement Point Library



Measurements and analyses in ArtemiS SUITE are based on the Measurement Point Library. This library generally lists all the measurement points that are measured or excited. In addition, a 3D model of the measurement object can be loaded into the Measurement Point Library. Such a 3D model (e.g., taken from a CAD program) then provides additional model points describing the geometry of the measurement more precisely.

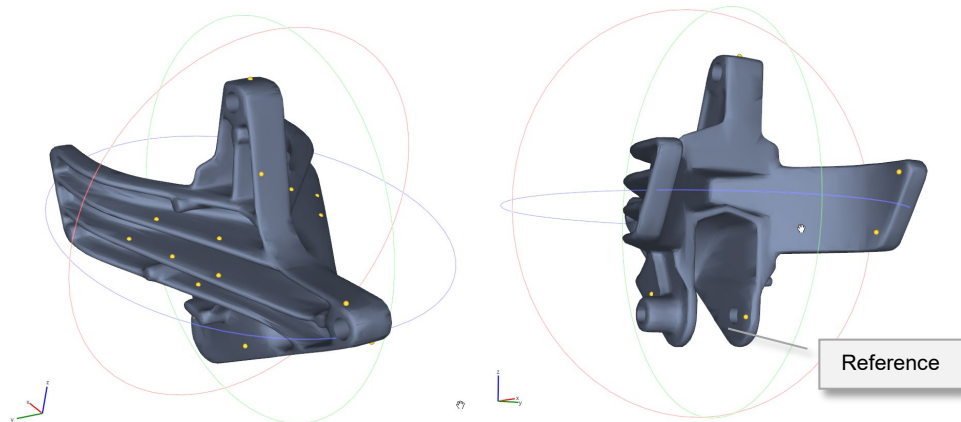


Figure 3: Representation of the measurement and model points of the torque arm in the Measurement Point Library

Impact hammer measurement



The measurements are carried out with the *Impact Measurement (Roving Hammer)* function. In this example, the impact hammer is used to excite the torque arm at 22 measurement points in two or three spatial directions. The system response is measured at a reference point in uniaxial direction. A total of 51 transfer functions are determined for this example. The impact hammer measurement of ArtemiS SUITE assists the user in setting parameters (e.g., sampling rate, window length and window function) and guides them, step by step, through all measurement points defined in the measurement point list.

Modal Analysis Project



After the measurements have been completed, the actual modal analysis is carried out in the Modal Analysis Project. In this project, the measurement-based transfer functions are analyzed and the modal parameters are determined. After opening the Modal Analysis Project for the first time, the Measurement Point Library and the corresponding transfer functions must be selected. Figure 4 shows the Modal Analysis Project for the torque arm example after the first opening.

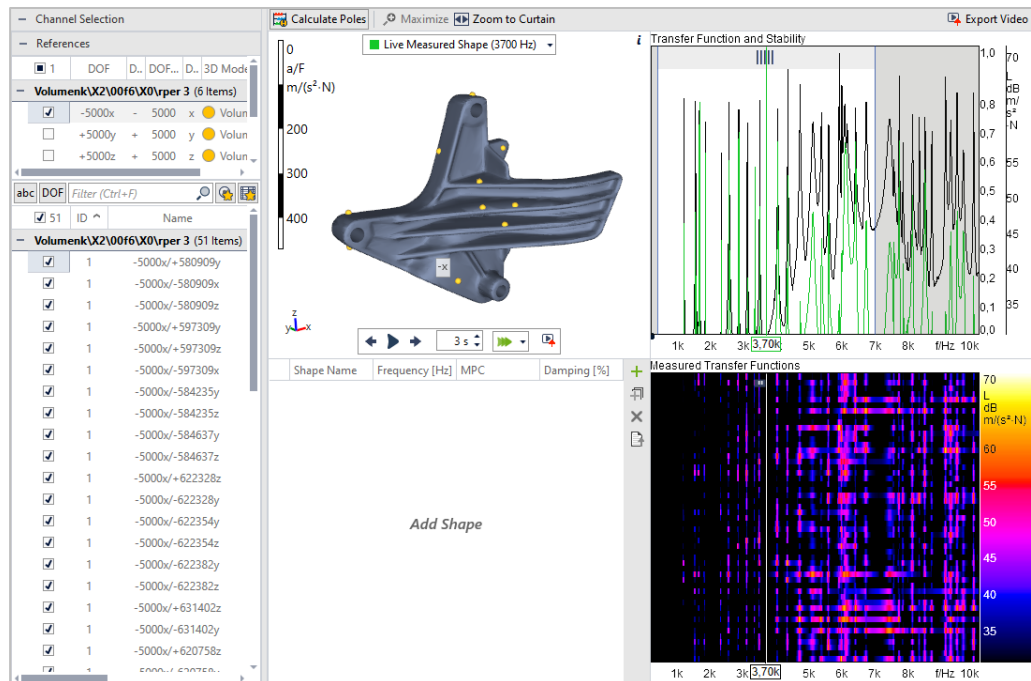


Figure 4: User interface modal analysis

Reference point(s) and frequency range

If data of a measurement with several reference points are available, these can be selected or deselected for the evaluation if necessary (Multi Reference). Afterwards, the frequency range for the modal analysis is defined. In this example, the frequency range from approximately 500 Hz to approximately 7000 Hz is to be investigated.

Determination of model size

Next, the model size for the pole calculation is set (*Highest Model Size*, see Figure 5). This parameter determines the maximum degree of the polynomial from whose coefficients the companion matrix is formed (see page 6). The formation of the companion matrix and the calculation of its eigenvalues is done iteratively with continuously increasing model size up to the maximum model size defined by the user. As an alternative to manual input by the user, the modal analysis project provides a neural network to determine these values automatically on the basis of the measurement data [4].

Pole Calculation	
Model Size Selection	Manual
Highest Model Size	100
Min. Stable Poles per Mode	5
Frequency Tolerance [Hz]	1
Damping Tolerance [%]	50

Figure 5: Parameters for pole calculation that were used for the present example

The higher this value, the more degrees of freedom are available and the more eigenvalues, also called poles, are found by the algorithm. However, a higher number of iterations can also cause undesired effects. If the model size is too large in relation to the real poles, there might also be poles found in the interfering parts (e.g., caused by noise) of the transfer functions. In addition, the calculation time will increase the more iterations are calculated and the more complex the model is.

Stability plot

The result for each model size (iteration) consisting of frequency and damping is passed to the stability plot. This plot shows how much the results vary over the iterations. For this purpose, the similarity of the poles is evaluated on the basis of parameters defined by the user. This results in the colored poles in the stability plot (see Figure 6). The colors of the dots provide information on their stability related to the different iterations:

- A red dot represents an unstable pole whose frequency and damping vary significantly at different iterations.
- If the pole is colored yellow, the frequency varies within specified tolerances at different iterations, but the damping shows significant deviations.
- A green dot indicates a stable pole whose frequency and damping remain within the specified tolerances at different iterations.

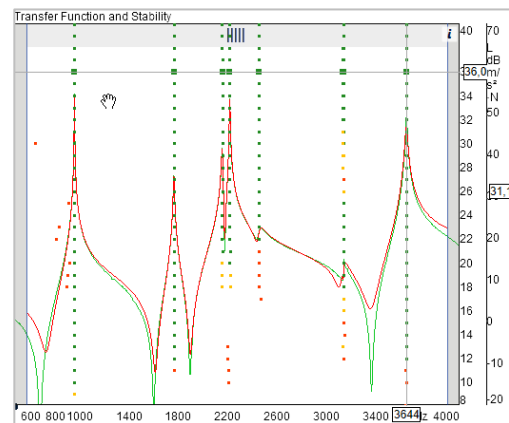


Figure 6: Stability plot with poles marked in color

Parameters for pole evaluation

The value *Minimum Stable Poles per Mode* allows the user to define when a pole is displayed as stable in the stabilization plot. The value determines the minimum number of iterations for which a pole must meet certain tolerances regarding frequency and damping. The user specifies the tolerances for the similarity of poles by using the values *Frequency tolerance* and *Damping Tolerance*. When checking the stability of the poles, the algorithm first examines the frequency deviation, and then the deviations in the damping.

Calculating the poles results in the stability plot shown in Figure 7.

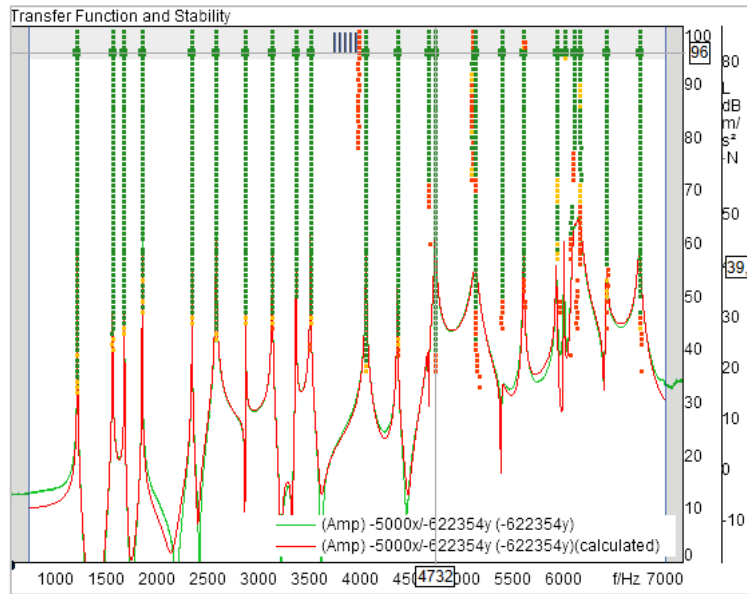


Figure 7: Stability plot for the torque arm modal analysis

Some of the poles found are shown enlarged. These mark the poles on the basis of which the curve fitting for approximating the transfer functions is performed. In addition to the measured transfer functions colored green, the transfer function approximated by the curve fitting is shown in red after the calculation.

Comparison of the measured and approximated transfer functions

In the first approach, the Modal Analysis Project first selects the poles from the iteration with the smallest deviations in pole frequencies for curve fitting. For this purpose, only stable poles are used. The horizontal auxiliary line makes it easier to read off the model size used in the process (see, for example, the horizontal line in Figure 7 labeled “96”). Scrolling through the various transfer functions allows a visual comparison between the approximated and the measured transfer functions (see Figure 8).

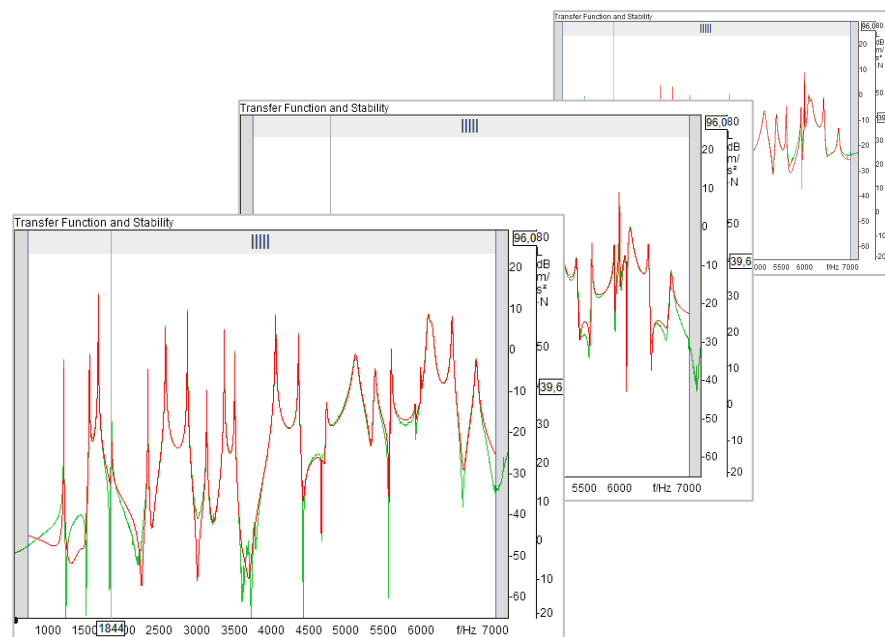


Figure 8: Comparison between the measured and the approximated transfer functions

The two curves should match well, especially at the maxima. If this is the case, the poles are well selected for curve fitting. If this is not the case, the selection of the poles needs to be adjusted. In most cases, the poles automatically selected by the Modal Analysis Project already provide a very good basis for curve fitting, thus ensuring a good match between the measured and the approximated transfer functions. Therefore, the preselected poles can usually be used without any adjustments.

Mode shape table

As a next step, the calculated eigenmodes with natural frequency and damping are therefore inserted into the mode shape table for further investigations. In addition to the modal parameters frequency and damping, the MPC value (Modal Phase Collinearity) for each eigenmode is also specified in this table (see Figure 9).

MPC value

The MPC value can be used as a tool to check the quality of the calculated eigenmodes. It describes the phase collinearity of the measured points at a certain frequency. The MPC value has a value range from 0 to 1, with 1 representing a high phase collinearity of the measurement points, i.e., the measurement points oscillate in phase or by 180° in opposite directions. The greater the phase differences of the measurement points, the more the value tends towards 0.

Shape Name	Frequency [Hz]	MPC	Damping [%]
Mode Shape (1207 Hz)	1207	0,795	0,0163
Mode Shape (1552 Hz)	1552	0,983	0,0741
Mode Shape (1670 Hz)	1670	0,864	0,0400
Mode Shape (1844 Hz)	1844	0,146	0,00596
Mode Shape (2336 Hz)	2336	0,701	0,02452
Mode Shape (2572 Hz)	2572	0,976	0,08432
Mode Shape (2866 Hz)	2866	0,849	0,02904
Mode Shape (3126 Hz)	3126	0,918	0,04888
Mode Shape (3360 Hz)	3360	0,824	0,03025
Mode Shape (3502 Hz)	3502	0,878	0,03500
Mode Shape (4050 Hz)	4050	0,915	0,07268
Mode Shape (4358 Hz)	4358	0,928	0,05048
Mode Shape (4665 Hz)	4665	0,848	0,13434
Mode Shape (4732 Hz)	4732	0,752	0,17173
Mode Shape (5127 Hz)	5127	0,874	0,58814
Mode Shape (5384 Hz)	5384	0,932	0,22904
Mode Shape (5603 Hz)	5603	0,822	0,06106
Mode Shape (5925 Hz)	5925	0,907	0,06632
Mode Shape (6000 Hz)	6000	0,777	0,01104
Mode Shape (6095 Hz)	6095	0,732	0,27475
Mode Shape (6153 Hz)	6153	0,456	0,45309
Mode Shape (6420 Hz)	6420	0,774	0,11744
Mode Shape (6743 Hz)	6743	0,907	0,23389

Figure 9: Mode shape table for torque arm modal analysis

Evaluation of the calculated eigenmodes

A high MPC value is a first indication that the extracted eigenmode exists in reality. On the other hand, a low MPC value is not an immediate exclusion criterion. Eigenmodes with a low MPC value should neither be deleted from the mode shape table without checking, nor be excluded from the subsequent evaluation. Instead, modes with low MPC values should be checked individually by the user. In the present example, the eigenmode at 1844 Hz has a very low MPC value of 0.148. This mode could not be optimally excited and measured with the measurement setup used. However, a resonance peak at this frequency can be detected in all transfer functions. With the selected poles, however, the phase progression for this eigenmode could not be approximated well for all transfer functions. This causes the motion of some points not to be in the correct phase relation to the other measurement points and thus leads to a

collapse of the MPC value. In order to check this, the phase can also be displayed in the stability plot. Figure 10 shows the amplitude and phase for the eigenmode at 1844 Hz as an example for a measurement point with a large phase deviation.

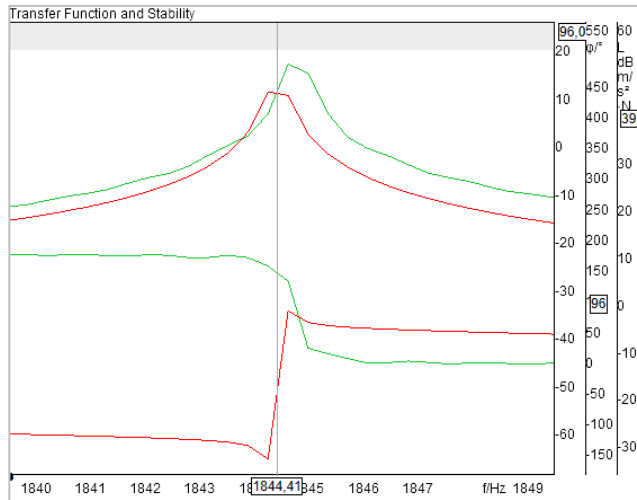


Figure 10: Calculated and approximated amplitude and phase at 1844 Hz

Since there is a peak at frequency 1844 Hz in all transfer functions, the corresponding frequency should be left as a natural frequency in the mode shape table. However, the calculated eigenmode of this frequency is subject to errors and must be examined critically. This is confirmed by a numerical simulation carried out later for the torque arm: this natural frequency is also found in the simulation, but the eigenmode has a different form.

The results of the modal analysis are the specifications in the mode shape table on natural frequency and damping for the calculated eigenmodes. These can be further investigated, for example, in the Shape Comparison Project.

Shape Comparison Project



In the Shape Comparison Project, the results from a mode shape table can either be compared with themselves or with the results of another calculation or simulation. The Shape Comparison Project automatically displays an (auto)MAC matrix for this comparison. The Modal Assurance Criterion (MAC) is a mathematical comparison method based on the evaluation of the eigenmodes in terms of the complex eigenvectors. Each matrix is a measure of the matching between the eigenvectors. If the eigenvectors match, the matrix element is equal to 1. If the two eigenvectors are orthogonal to each other, it is 0. The (auto)MAC matrix compares the eigenvectors of a modal model with itself and has as many columns and rows as there are eigenvectors in the loaded mode shape table.

Evaluation of the (auto)MAC matrix

In an (auto)MAC matrix, the values of the diagonals always have the value 1, because here the respective eigenvector is compared with itself. The values off the diagonals should be close to 0. If high values occur off the diagonals, this is an indication that the modal model describes reality inadequately. The reason for this is an insufficient geometric resolution of the measurement grid for the measurement task. As a result, information is not captured and the respective eigenvectors are incomplete and too similar. If a numerical model is available, the (auto)MAC matrix can be used to determine the optimal geometric resolution of the measurement grid before the measurement. By iteratively evaluating the (auto)MAC for models with an increasing number of virtual measurement points, it is possible to determine the resolution that is necessary to capture all required information.

The (auto)MAC matrix for the torque arm eigenmodes is shown in Figure 11. It shows the targeted high values in the diagonal. The other values are sufficiently low.

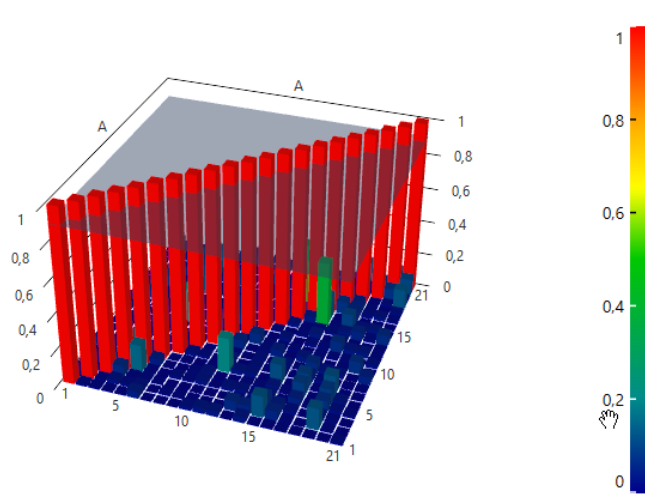


Figure 11: (auto)MAC matrix for the calculated torque arm eigenmodes

Comparison with results from simulation

Next, the results from the experimental modal analysis in the Shape Comparison Project can be compared with the results of a numerical simulation, for example. For this purpose, in addition to the mode shape table from the Modal Analysis Project, the one from a numerical simulation is loaded and the MAC matrix is calculated. The result is shown in Figure 12. The MAC values decreasing with higher frequencies are caused by the fact that with increasing frequencies small differences in the positioning of the real and virtual sensors have an increasing influence. Furthermore, as expected, a worse matching can be seen at the frequency of 1844 Hz, because the corresponding eigenmode could not be accurately depicted by the modal model (cf. page 11f).

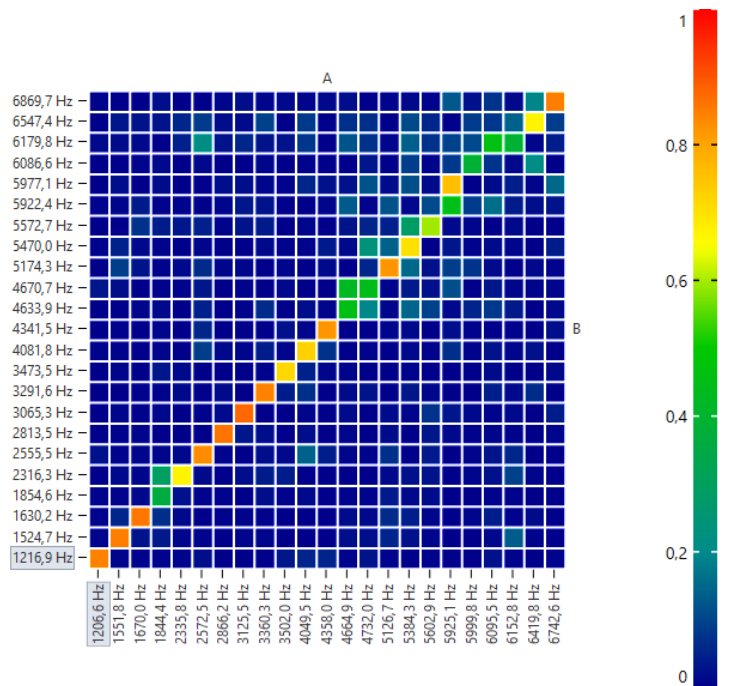
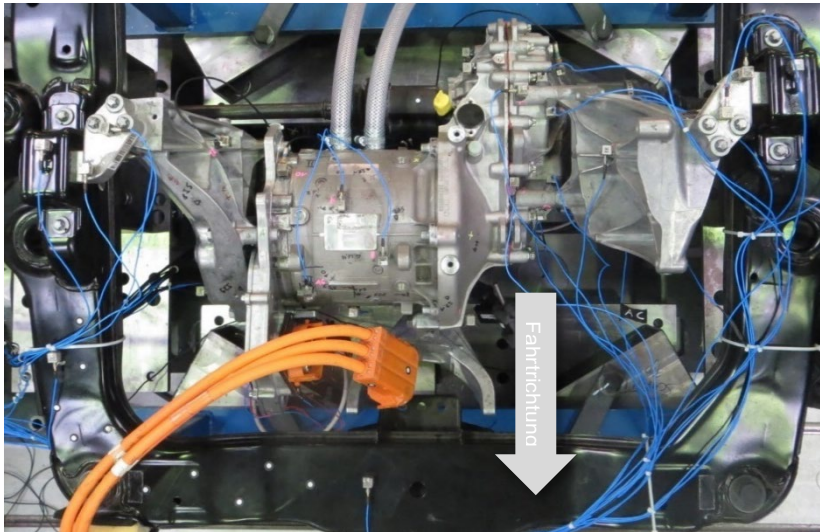


Figure 12: MAC matrix for the comparison of experimental (A) and numerical data (B)

Overall, the comparison shows a good matching between the results of the experimental modal analysis and those of the simulation performed with the finite element method.

EXAMPLE 2: POWERTRAIN OF AN ELECTRIC VEHICLE

Measurement setup powertrain This section briefly presents the implementation and results of a modal analysis for the powertrain of an electrically driven passenger car. In this case, the focus of the



experimental modal analysis was to gain knowledge about the structural dynamic properties of the powertrain. Figure 13 shows the powertrain mounted on the test rig from above.

Figure 13: Powertrain of the E-vehicle mounted on the test rig

The system response of the powertrain was recorded with 37 triaxial accelerometers. The excitation was done with an impact hammer (steel tip) at four different points normal to the surface. The 3D model used and the different sensor positions are shown in Figure 14.

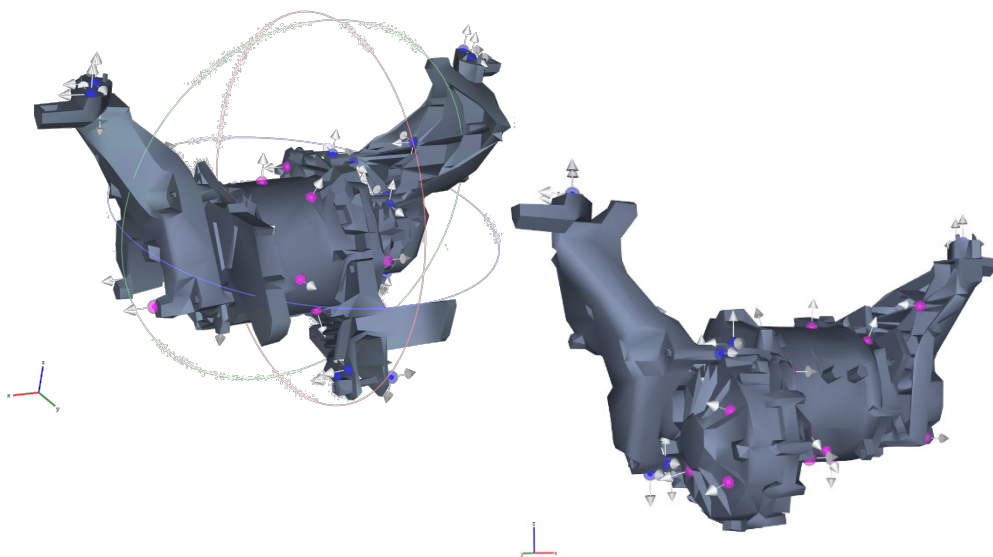


Figure 14: 3D model of the powertrain including sensor positions (left: front view, right: rear view)

Thus, for this example, a much higher number of channels was recorded than in the torque arm example. This, combined with the four reference points, results in a higher number of transfer functions. The modal analysis of the powertrain is thus significantly more complex.

Evaluation in the Modal Analysis Project

The evaluation of the measurements in the Modal Analysis Project was carried out in several steps: pole calculations were performed for different frequency ranges with parameter settings matched to them. In addition, the poles were determined using transfer functions with different reference points.

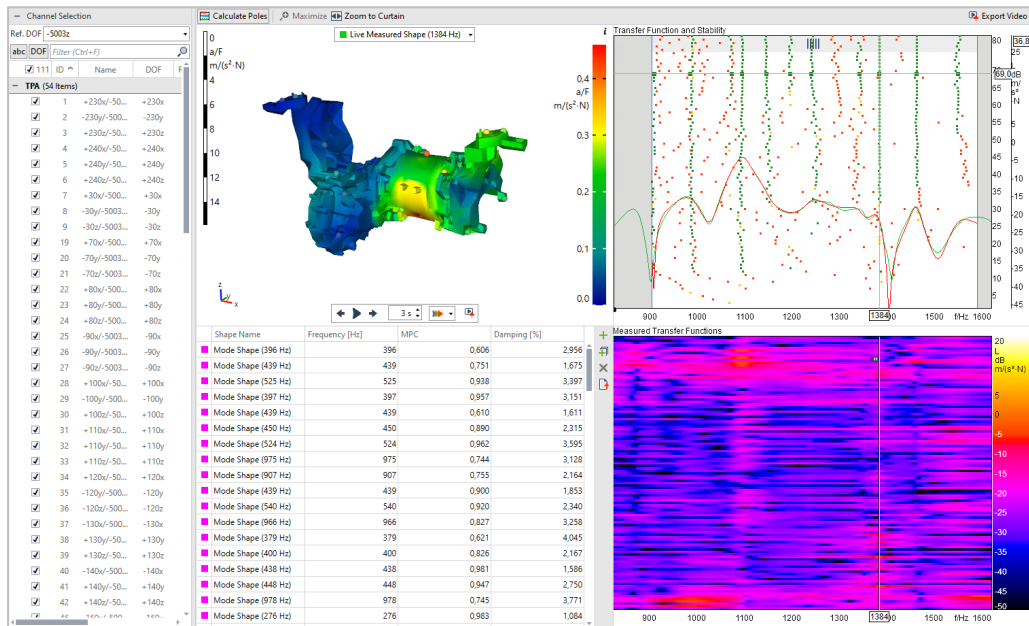


Figure 15: Analysis of the powertrain in the Modal Analysis Project

Calculated eigenmodes

Figure 16 shows three of the eigenmodes found. While the eigenmode at 276 Hz is dominated by the motion of the motor support arms, the eigenmode at 1352 Hz shows an ovalization of the electric motor housing. At 2507 Hz, the motion of the torque arm dominates. Thus, modes were extracted in a wide frequency range that are dominated by different parts of the structure.

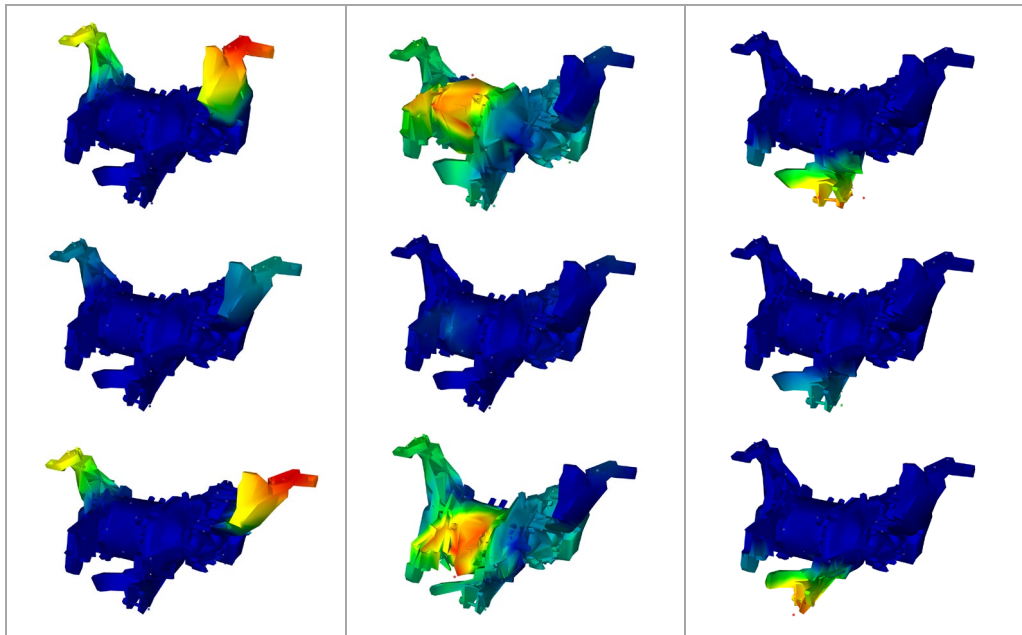


Figure 16: Eigenmodes of the powertrain (left: 276 Hz, middle: 1352 Hz, right: 2553 Hz rear view)

The results of these calculations were collected in the mode shape table of the Modal Analysis Project and, taking into account various evaluation factors (e.g., MPC values, comparison of the measured and approximated transfer functions, consideration of the calculated eigenmodes), a selection of 31 eigenmodes for a frequency range of 230-2600 Hz was made.

Evaluation in the Shape Comparison Project

The (auto)MAC matrix was then determined for this selection in the Shape Comparison Project (see Figure 17). It shows the targeted high values on the diagonal. The other values are sufficiently low.

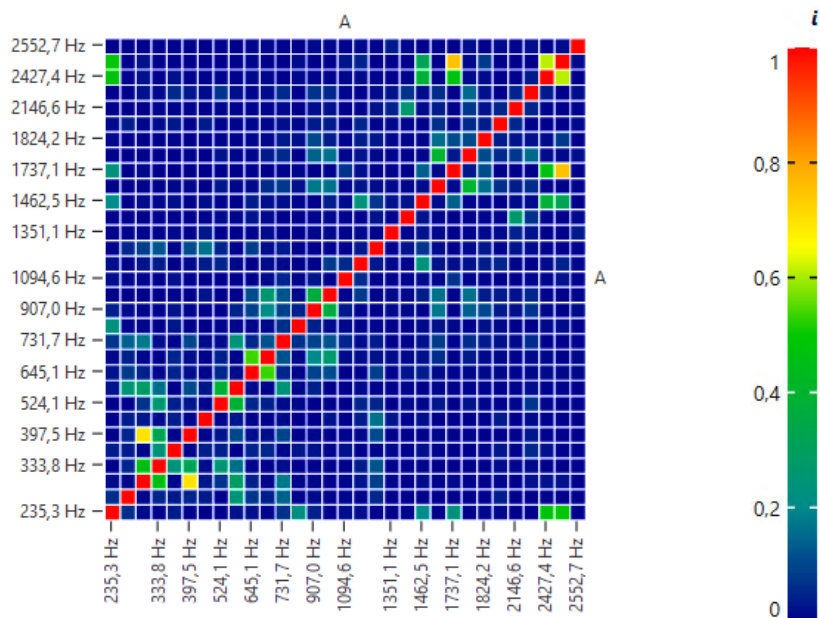


Figure 17: (auto)MAC matrix of the eigenmodes from the modal analysis in the Shape Comparison Project

The measurement data shown were recorded as part of a sponsored research project on a test rig at the Institute for Machine Elements and System Development at RWTH Aachen University, Germany.

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4 REFERENCES

- [1] P. Guillaume et al.
“A poly-reference implementation of the least-squares complex frequency-domain estimator”
Proceedings of the International Modal Analysis Conference, February 2003
- [2] P. Guillaume, P. Verboven and S. Vanlanduit.
“Frequency-domain maximum likelihood identification of modal parameters with confidence intervals”
Proceedings of the International Conference on Noise and Vibration Engineering, September 1998
- [3] M. El-Kafafy, P. Guillaume
“Modal Parameter Estimation of Structures with Overdamped Poles”,
Proceedings of the International Conference on Noise and Vibration Engineering, January 2010
- [4] T. Kamper, M. Wegerhoff, T. Lobato, R. Sottek
„Simplifying the Extraction of Modal Parameter from Test Data using Neural Network”
Proceedings of the 48th Annual Conference on Acoustics DAGA, March 2022