HEAD acoustics

APPLICATION NOTE

Information on this document

Contents

This document is the first in a series of four application notes on structural dynamics. The document includes some notes on the benefits of structural dynamic investigations as well as a glossary of terms, the understanding of which will form the basis for the other parts in this series.

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Target group

This document serves as an introduction to structural dynamics and is intended for acousticians who would like to gain an insight into this subject and familiarize themselves with the basic concepts.

Questions?

Do you have questions? Your feedback is appreciated!
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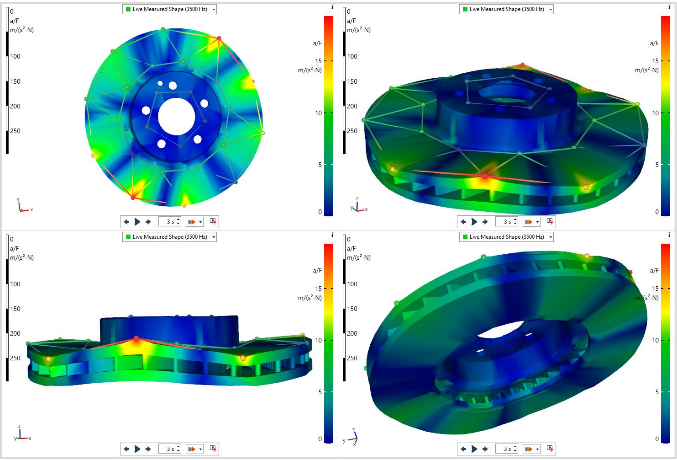
Structural Dynamics – Part 1 (Glossary)

1. Benefits of Structural Dynamic Investigations

Motivation

Knowing the dynamic response of structures allows to optimize the mechanical vibrations of these structures by constructive measures. In this way, for example, the structure-borne sound transmission and radiation behavior of a structure can be optimized. In addition, by reducing the vibrations the load on the structure can be lowered, thus increasing the service life of a component or machine.



Insights from structural dynamics

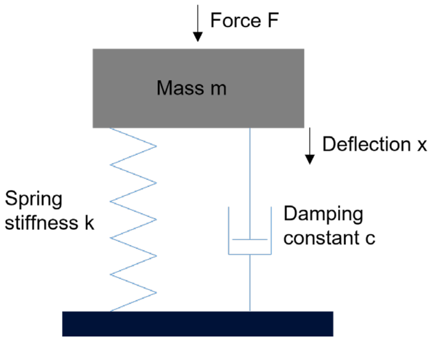
Important insights can be gained by analyzing the structural dynamics:

- how a structure vibrates during excitation under real conditions
- the sound radiation to be expected from the surfaces of a structure (e.g., a machine) and ways to avoid it
- find the cause of sound and vibration problems and optimize the transmission paths
- how to optimize the structural system for sound and vibration applications
- the expected dynamic stress and associated material fatigue
- locate weaknesses in the structure with regard to dynamic stress and evaluate solutions
  (The aim is to adjust a structure in such a way that its natural frequencies do not coincide with the excitation frequencies, thus reducing or even eliminating dynamic influences.)
- predict the effects of modifications and bypass the trial-and-error approach to solving existing vibration problems, thus reducing time and cost of implementing design changes
- verify and adjust the mathematical models of the structure

2. Glossary

Degrees of freedom DOF

- **DOF** (degree of freedom): In general, DOF is referred to as the possible independent movement of a structural point. In structural dynamics, a DOF defines a measuring point on a structure including its measuring direction (e.g., x, y, or z direction).

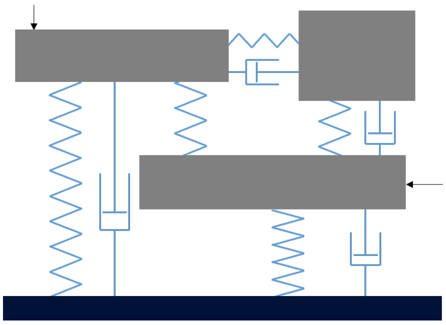


- **SDOF** (single degree of freedom): The single degree of freedom is formed by a mass m, a spring with a spring stiffness k and a damper with a damping constant c. If the mass is moved by a force F from its rest position by a deflection x, it can only move in one degree of freedom. The forced, damped vibration of a single degree of freedom is described by the following equation of motion:

  $$F(t) = m\ddot{x}(t) + c\dot{x}(t) + kx(t)$$

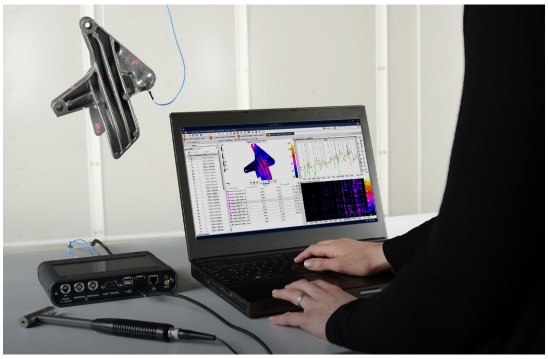
  An SDOF has one degree of freedom and is therefore also called a single degree of freedom system. Such a system has one natural frequency and one corresponding eigenmode. The SDOF system forms the basis for systems with more than one degree of freedom. The term SDOF also refers to a group of curve-fitting methods.

- **MDOF** (multiple degree of freedom): Linear multiple degree of freedom systems can be described as the sum of multiple SDOF systems. There is usually a mutual influence of the individual SDOF systems. The number of natural frequencies or eigenmodes of an MDOF system cannot exceed the number of degrees of freedom.
  The term MDOF also refers to a group of curve-fitting methods.



Procedure with structural analysis

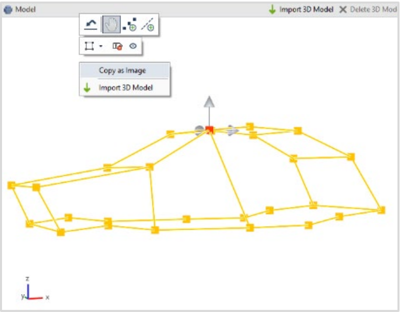
- **EMA** (experimental modal analysis, also known as modal tests): EMA is a method for analyzing the dynamic properties of linear, time-invariant structures. In EMA, the structure is excited with an impact hammer or shaker(s) and the structure's response is measured with sensors (usually accelerometers). The acquired data and further analyses allow the determination of the modal quantities (natural frequencies, damping, eigenmodes).



- **Curve Fitting**: Curve Fitting is a mathematical process for generating a curve that matches a target curve as closely as possible. In structural analysis, curve fitting refers to the recreation of a transfer function. Natural frequencies and dampings of the structure are extracted. Various methods have been developed for curve fitting. Using curve fitting for several transfer functions at the same time, the natural frequencies and the corresponding eigenmodes can be determined.

- **OMA** (operational modal analysis or output-only modal analysis): OMA is a method for determining modal parameters in which structural excitations present in operation are used for modal analysis. It is used, for example, if it is not possible to apply sufficient excitation energy to the structure by using a hammer or shaker. Furthermore, OMA can be used if the structure to be investigated needs to be analyzed in assembled condition (e.g., an installed powertrain). Also with OMA, specific software is used to extract modal quantities. The problem with OMA is to achieve a broadband uniform excitation of the structure without tonal components due to operating forces. A broadband form of excitation is the mathematical prerequisite for calculating the modal parameters in case of unknown excitation.

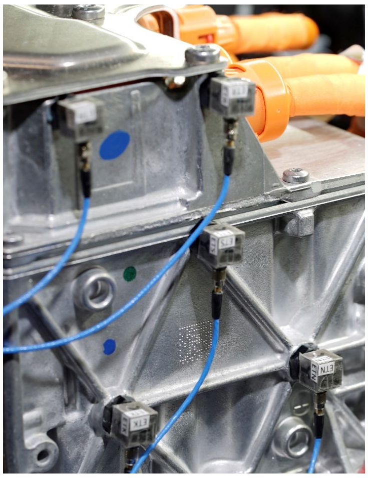
- **ODS analysis** (operational deflection shape analysis): An operational deflection shape analysis examines the vibration behavior of a structure in operating condition. For this purpose, the vibration of the structure during operation is recorded at numerous points (the excitation forces are not explicitly measured). Modal parameters such as natural frequencies and modal damping cannot be determined by operational deflection shape analysis. The deflection shapes determined by ODS usually do not correspond to the eigenmodes, but reflect the response behavior of the structure to an (unknown) excitation induced by operation. The deflection shapes are usually composed of several eigenmodes.
  Using a visualization model, the operating deflection shapes can be displayed in animated form as a function of time or frequency.



- **FEM** (finite element method): The finite element method is a widely used numerical method for solving differential equations. With this method, the structure is divided into finite elements – i.e., small finite areas – and a model of the structure is created on the computer. FEM can be used, for example, to perform numerical modal analyses and calculate transfer functions.

Measurement methods

- **SISO** (single input - single output): SISO measurements include modal analysis with a hammer or shaker (single input) and an accelerometer (single output). This measurement method is suitable for smaller structures; it is a very cost-effective measurement, since little measurement equipment is needed.
- **SIMO** (single input - multiple outputs): SIMO measurements include modal analysis with a hammer or a shaker (single input) and multiple accelerometers (multiple outputs). The measurement time can be significantly reduced by using multiple accelerometers.
- **MIMO** (multiple inputs - multiple outputs): MIMO measurements include, for example, modal analysis with multiple shakers (multiple inputs) and multiple accelerometers (multiple outputs). An advantage of MIMO measurements is that the injected energy is distributed over multiple locations in the structure. This provides a more uniform vibration behavior of the structure, especially for large and complex structures and those with strong damping. MIMO measurements can be used, for example, if the test structure has closely spaced or coupled modes.



- **Roving Hammer**: In the roving hammer method, the structure is excited with a hammer at various points. The structural response is measured with one or more accelerometers at a fixed position in each case. This has the advantage that the mass influence of the sensors does not change between the individual measurements. The roving hammer method is particularly suitable for smaller structures and can be performed with relatively little measurement equipment. The roving hammer method can only be used if all required positions can be struck with the impact hammer.

- **Roving Accelerometer**: In the roving accelerometer method, the structure is excited with a hammer or shaker at a fixed position. The structural response is measured with one or more accelerometers, the position(s) of which is/are changed after each measurement. If several accelerometers are used, data acquisition can be accelerated considerably, since the transfer function for several measurement points can be determined simultaneously with one measurement. The roving accelerometer measurement should be used if not all points can be easily reached for excitation with the impact hammer. On the other hand, when evaluating the data, it has to be taken into account that moving the sensor(s) will change the mass influence on the structure and can lead to differences between the measurements.

- **Roving Hammer/Roving Accelerometer combination**: The roving hammer method and the roving accelerometer method can be combined to excite the structure at different points. This is particularly useful if not all modes of the structure can be excited at one force transmission point. This method is also suitable for separating strongly coupled modes.

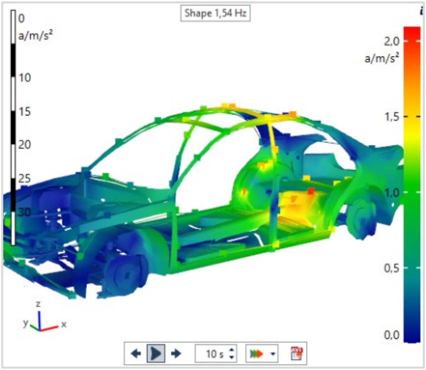
- **Multi Reference**: Some applications require more than one reference point. In such a case, the measurement is referred to as a multi-reference measurement. This type of measurement is necessary, for example, if not all modes of the structure can be excited by the force introduction at one point and several introduction points are required for the excitation. Another application for multi-reference measurement involves structures having several modes with closely spaced natural frequencies. This may occur, for example, with some symmetrical structures.
  The combination roving hammer / roving accelerometer belongs to the multi-reference measurements category, and so does the MIMO measurement.

Other commonly used words

- **Natural frequency**: The natural frequency f_0 is the frequency at which an oscillating single degree of freedom system oscillates after a single, broadband excitation (free oscillation).
  In the case of a periodic excitation with the frequency f_a, the system oscillates with the frequency f_a (forced oscillation). If the frequency f_a of the excitation is the same as the system's natural frequency f_0, the system responds with high amplitudes (resonance case).
  A structure's natural frequency is an inherent property and depends on the structure's geometry, composition and material properties.

- **Eigenmode**: The eigenmode is a theoretical deflection shape a system shows under excitation with natural frequency f_0. An eigenmode is an inherent property of the system. Each natural frequency has a specific deflection shape. If a system has several natural frequencies, it also has several eigenmodes. One of the most important properties of eigenmodes is orthogonality, i.e., the modes are linearly independent of each other.

- **Deflection shape**: A dynamic, time-varying excitation induces an oscillatory structure to oscillate with a certain deflection shape. The deflection shape is described by the movement of the structure points in relation to each other. It depends on the dynamic properties of the structure and on the excitation. Each deflection shape is a linear combination made up of the individual eigenmodes. The relative share of the individual eigenmodes in the overall vibration depends on the frequency distribution and the location of the excitation. Even an excitation with only one frequency results in a deflection shape composed of a combination of different eigenmodes.



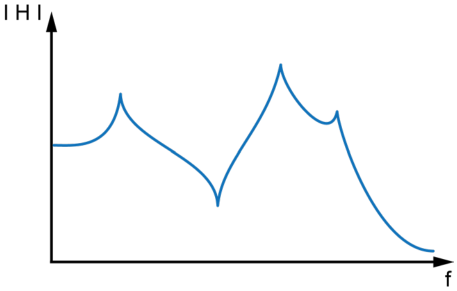
- **Rigid body mode**: In rigid body mode, the structure remains rigid and shifts as a whole. Thus, the motion occurs without deformations, strains or stresses. A free structure in space has six rigid body modes: translational motion in the x, y and z direction and rotational motion around the x, y and z axes. The natural frequency of the rigid body mode is 0 Hz. In a real measurement, the actual frequency of the rigid body mode may be higher than 0 Hz, for example, due to the elastic support/suspension.

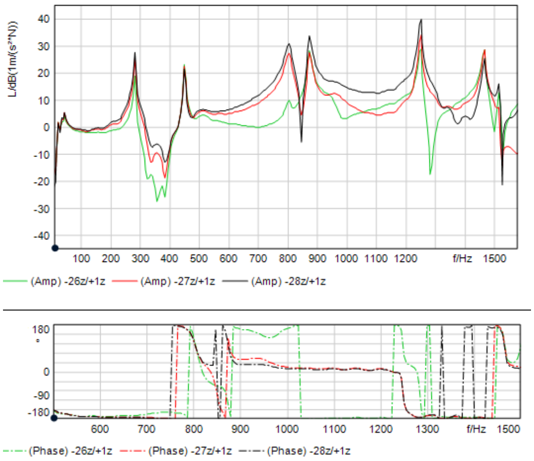
- **Flexible mode**: In flexible mode, a structure is deformed and oscillates around a rest state. The natural frequencies of flexible modes are higher than 0 Hz.

- **Frequency response function, FRF or transfer function)**: The frequency response function describes the frequency-dependent ratio of the vibration response of a structure to the excitation. For structures that behave linearly, the system response can be calculated directly from the excitation signal and the frequency response function.

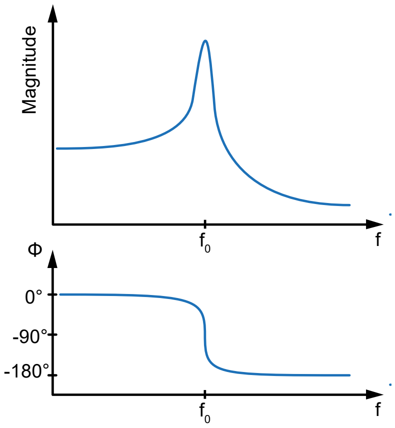
  A frequency response function contains information on the natural frequencies and modal damping, and a set of frequency response functions also includes information on the corresponding eigenmodes of the system. The frequency response functions provide information on the dynamics of the system: mass, rigidity and damping or natural frequencies, eigenmodes and modal damping.

  The frequency response function is a complex-valued function that can be represented by real and imaginary parts or magnitude and phase. In the so-called Bode diagram, the magnitude and phase of the frequency response function are shown as a function of frequency.





- **Phase**: The time shift between the input signal (load force) and the system response changes depending on the frequency. This shift can be represented in the form of the phase response (phase over frequency). The phase response provides information on the relative direction of oscillation, the mode shape and existing natural frequencies. The system response below the natural frequency is in phase with the excitation. When the natural frequency f₀ is reached, the phase curve tilts. Above the natural frequency, the input and output signals are shifted by 180°.



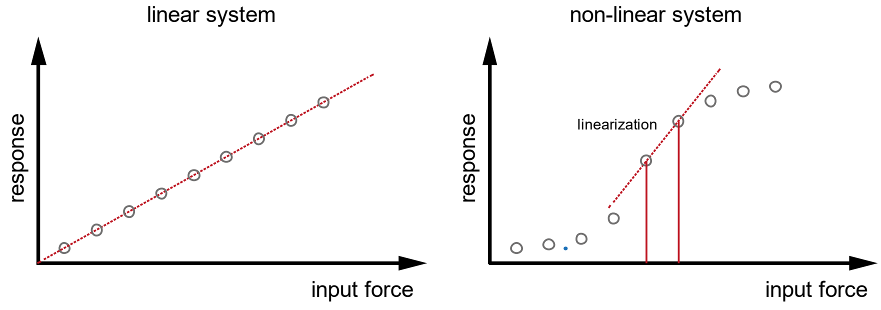
- **Coherence**: Coherence describes the linear dependence between two signals in the frequency domain (e.g., between excitation signal and system response). The coherence between the input and the output signal allows for the identification of response components that are not causally based on the input signal, but were caused by additional external excitations. It also allows for the detection of non-linearities of a system.

  The coherence can assume values between 0 and 1 and is plotted over frequency. The actual coherence values depend very much on the practical application. In the realm of natural frequencies, the coherence should be very close to 1. With anti-

resonances, the signal-to-noise ratio is poor, and coherence breaks down at these points. However, this does not allow any conclusions to be drawn about the quality of the measurement.
When calculating the coherence, it must be taken into account that meaningful results can only be obtained by averaging several measurements. Without averaging, the coherence assumes the value 1 over the entire frequency range.
If the system was excited with several input sources (e.g., with multiple shakers), the partial or multiple coherence must be determined in order to evaluate the quality of the measurement.

  - Partial coherence describes the linearity between a single input signal and the output signal. The linear influence of other input channels is removed in the calculation. This presupposes that the input channels are as uncorrelated as possible.
  - Multiple coherence provides a statement on the common linear dependence between several input signals and the output signal.

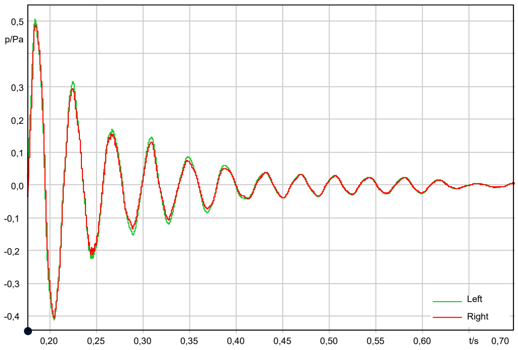
- **Linearity**: In linear systems, the output signal changes proportionally to the excitation signal. As a consequence, the superposition principle applies to linear systems.
If the properties of the system change, for example, due to deformation or temperature influences, it is a non-linear system. Non-linear properties become more important especially at high vibration amplitudes.
A system’s linearity can be checked by calculating the coherence at different levels of excitation by the impact hammer. With non-linear structures, the coherence breaks down over a broad band.
To investigate non-linear systems, measurements can be made for a specific force amplitude for which the system can be assumed to be linear (sectional linearization).



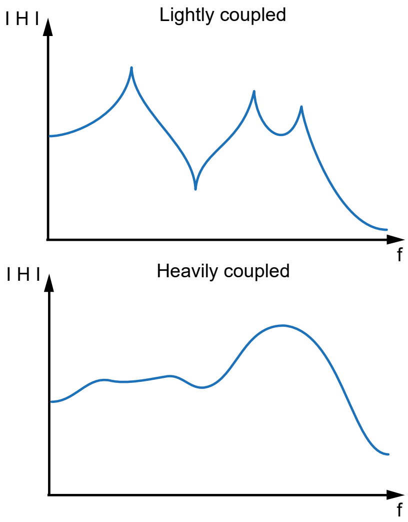
- **Reciprocity**: In a reciprocal system, the frequency response function between two points is independent of the point at which excitation occurs or the system response is measured. This means that excitation at point A and measurement of the response at point B in a reciprocal system results in the same frequency response function as excitation at point B and measurement of the response at point A.

- **Time invariance**: Time-invariant systems always respond in the same way, regardless of time. This means that the time delay of an input signal leads to a corresponding delay in the system response, without, however, influencing the form of the output signal.

- **Damping:** In damped, oscillating systems, the amplitude of the oscillation decreases over time, provided that no further energy is supplied to the system. In addition to the mass and the stiffness as well as their distribution, damping determines the dynamic response of an oscillatory structure.



- **Modal coupling:** Complex systems with multiple degrees of freedom (MDOF systems) can be described as the sum of several single degree-of-freedom systems (SDOF systems). Usually, the individual SDOF systems influence each other. This effect is called modal coupling. The degree of modal coupling indicates to what extent the system response at a given natural frequency is influenced by other modes. Basically, two types of modal coupling are distinguished:
    - Lightly coupled modes: the influence of adjacent modes on each other is small, and the individual modes are easy to identify in the transfer function. Structures with this property respond similarly to an SDOF system in the range of their natural frequencies.
    - Heavily coupled modes: the modes influence each other and are difficult or even impossible to identify in the transfer function. The structure is strongly damped and/or shows a high mode density. Determination of the modal parameters is possible, but proves to be relatively difficult.



➡ *Proceed to the [second application note on structural analysis] providing an introduction into experimental modal analysis*